Are We Prepared To Accept The Reality ?

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August 1, 2020

Abstract

In this lecture an attempt is being made to convince the world, especially we Indians that in reality Bharat, the Greater India, is the origin of mathematics. This will correct what most of the historians of Mathematics have claimed in their books, and we have accepted it religiously.

We will also try to reassure that Mathematics as well as Proof cannot be defined. Further, we shall discuss features of Upapattis (word in Pali, Sanskrit, and Marathi languages) in Indian Mathematics.

Bibhutibhushan Datta (1888-1958, India), *The Science of the Sulbas: A Study in Early Hindu Geometry*, *Calcutta University Press*, **Calcutta, 1932.**

Page 27 reads: The reference to the sacrificial altars and their construction is found as early as the Rg-Veda Samahita (before 3000 BC). ... It seems that the problem of the squaring of the circle and the theorem of the square of the hypotenuse are as old in India as the time of Rg-Veda. They might be older still.

Four Vedas (means wisdom, knowledge or vision), *Rigveda*, *Samaveda*, *Yajurveda*, **and** *Atharvaveda* **were codified by Krishna Dwaipayana or Sage Veda Vyasa (born 3374 BC) along with his disciples Jaimani, Paila, Sumanthu, and Vaisampayana. Each Veda consists of four parts – the Samhitas (hymns), the Brahmanas (rituals), the Aranyakas (theologies) and the Upanishads (philosophies).**

Arthur Schopenhauer (1788-1860), One of the greatest German philosophers and writers: In the whole world there is no study so beneficial and so elevating as that of the Upanishads. It has been the solace of my life; and it will be the solace of my death. They are the product of the highest wisdom. “Vedas are the most rewarding and the most elevating book which can be possible in the world.”

Niels Bohr (1856-1962), Nobel Laureate in Physics (1922) for his theory of Atomic structure: I go into the Upanishads to ask questions.

Stephen Knapp, The Secrete Teachings of the Vedas, Jaico Publishing House, Mumbai, 1993.

Stephen Knapp, Proof of Vedic Culture’s Global Existence Paperback, Booksurge, Charleston, 2009.

During Vedic period (at least 5000 years ago) it was known that the distances of the Moon and Sun from the Earth are 108 (which is a sacred number in several eastern religions and cults) times the diameters of these heavenly bodies, respectively, (the observed values are 110.6 for the Moon and 107.6 for the Sun). Hindus also discovered what is known as the *precession of the Equinoxes* and their calculation such an occurrence takes place every 25827 years, our modern science after labors of hundreds of years has simply proved them to be correct.

Jean Sylvain Bailey (1736-93), Great French astronomer and politician, noted for his computation of an orbit for Halley's Comet. The motion of the stars calculated by the Hindus before some 4500 years vary not even a single minute from the modern tables of Cassine and Meyer.

John Archibald Wheeler (1911-2008), Eminent American physicist, the first one to be involved in the theoretical development of the Atomic bomb and the first to coin the term 'Black Hole' who later occupies the chair that was held by Albert Einstein: I like to think that someone will trace how the deepest thinking of India made its way to Greece and from there to the philosophy of our times.

Sulbasutras

The meaning of the word *sulv* is to measure, and geometry in ancient India came to be known by the name *sulba* or *sulva*. The Sulbasutras are the appendices to four Vedas. Only seven Sulbasutras are extant, named for the sages who wrote them: Apastamba, Baudhayana (born 3200 BC), Katyayana, Manava, Maitrayana, Varaha, and Vidhula. The four major Sulbasutras, which are mathematically the most significant, are those composed by Baudhayana, Manava, Apastamba, and Katyayana. These Sulbasutras contain a large number of geometric constructions for squares, rectangles, parallelograms and trapezia; the problem of solving quadratic equations of the form $ax^2 + bx + c = 0$; several examples of arithmetic and geometric progressions; a method for dividing a segment into 7 equal parts; solutions of first degree indeterminate equations; and (without any proofs) remarkable approximations of $\sqrt{2}$ and π.

In the book *India in Greece* published in 1852, in England, by the Greek historian Edward Pococke reports that Pythagoras, who taught Buddhist philosophy, was a great missionary. His name indicates his office and position; Pythagoras in English is equivalent to putha–gorus in Greek and Budha–guru in Sanskrit, which implies that he was a Buddhist spiritual leader. Note that Lord Gautama Buddha was during (1887-1807 BC), historians have misled the world by claiming that he flourished around (450 BC).

Francois Marie Arouet Voltaire (1694-1778), one of the greatest French writers and philosophers: "I am convinced that everything has come down to us from the banks of Ganga–Astronomy, Astrology, and Spiritualism. Pythagoras went from Samos to Ganga 2600 years ago to learn Geometry. He would not have undertaken this journey had the reputation of the Indian science had not been established before."

Thomas Stearns Eliot (1888-1965), American-British poet, Nobel Laureate (1948): "I am convinced that everything has come down to us from the banks of the Ganga–Astronomy, Astrology, Spiritualism, etc. It is very important to note that some 2,500 years ago at the least Pythagoras went from Samos to the Ganga to learn Geometry but he would certainly not have undertaken such a strange journey had the reputation of the Brahmins' science not been long established in Europe."

According to historians Pythagoras also went to Egypt and Babylonia in search of knowledge. Some prominent historians of mathematics: Bartel Leendert van der Waerden (1903-1996), Dirk Jan Struik (1894-2000), and Sir Thomas Little Heath (1861-1940) have suggested that Egyptians had no knowledge of Pythagorean Theorem, which is not true, see Berlin Papyrus 6619, and Cairo Mathematical Papyrus, unearth in 1938 and first examined in 1962. Of course, there is a sufficient evidence that Pythagorean Theorem was known to Mesopotamian, see "YBC 7289", and "Plimpton 322", written between 1790 and 1750 BC. However, above two quotations strongly suggest that Pythagoras learnt about Pythagorean Theorem and Triples probably in India. For their origin, patterns, extensions, astonishing directions, and open problems, see

- **R.P. Agarwal**, Pythagorean theorem before and after Pythagoras, *Advanced Studies in Contemporary Mathematics,* **30**(2020), 357–389.
- **R.P. Agarwal**, Pythagorean triples before and after Pythagoras, *computation* **2020**(2020), 8, pages 36, 62; doi:10.3390/computation8030062.

India is considered the Land of Numbers. She has produced several leading Number Theorists such as Srinivasa Ramanujan (1887-1920), FRS (1918). He was one of the greatest Intuitionalist. In the world, although there are about four thousand languages of which several hundred are widespread, there are only several dozen alphabets and writing systems that represent them; however, we can safely say that there is but one single place–value system that uses zero and nine numerals, symbolically $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$, by which any number can be expressed and understood rather easily. This gift of India has truly united the universe in the language of numbers. It is worth noting that there is hardly any aspect of our life in which numbers do not play a significant–though generally hidden–part.

In 1875, George Thibaut (1848–1914) translated a large portion of the Sulvasutras, which showed that the Indian priests possessed significant mathematical knowledge. Thibaut was a Sanskrit scholar and his principal objective was to make the mathematical knowledge of the Vedic Indians available to the learned world. He firmly believed that Hindus had knowledge of incommensurability (irrationality), in particular, of $\sqrt{2}$. In fact, in Apastamba there is a discussion of the irrationality of π.

Acharya Charak and Acharya Sudhrut (before 600 BC). Charak has been crowned the Father of Medicine. His *Charak Samhita* is considered to be an encyclopedia of Ayurveda, which means science of life. In this work there is also an interest in permutation and combination. His treatise *Sushrut Samhita* states that 63 combinations can be made out of 6 different rasa (tastes)–bitter, sour, salty, astringent, sweet, and hot–by taking the rasa one at a time, two at a time, three at a time and so on. This solution of 63 combinations can be checked:

$$ {}_6C_1 + {}_6C_2 + {}_6C_3 + {}_6C_4 + {}_6C_5 + {}_6C_6 = 6 + 15 + 20 + 15 + 6 + 1 = 63. $$

Pingala (about 500 BC) was the author of the *Chhandah-shastra*, the Sanskrit book on meters, or long syllables. Pingala was the younger brother of the great grammarian Panini, the author of the famous treatise *Asthadhyayi* of the fifth century BC. Pingala presented the first known description of a binary numeral system. He described the binary numeral system in connection with the listing of Vedic meters with short and long syllables. His work also contains the basic ideas of *maatraameru* and *meruprastaara*, concepts that are inappropriately now known as Fibonacci numbers and Pascal's triangle, respectively. His book Chhandah-shastra contains the first known use of zero, indicated by a dot (·). In this book he also explored the relationship between combinatorics and musical theory, which was later reproduced by Mersenne. Around the end of the tenth century, Halayudha (about 975) produced a commentary on Pingala's Chhandah-shastra in which he introduced a pictorial representation of different combinations of sounds, enabling them to be read off directly. Halayudha's meruprastaara (or pyramidal arrangement) provides the binomial expansion of $(a+b)^n$, where $n = 1, 2, 3, \ldots$.

Pierre Simon De Laplace (1749-1827) French greatest mathematician, philosopher and astronomer: It is India that gave us the ingenious method of expressing all numbers by ten symbols, each receiving a value of position as well as an absolute value, a profound and important idea which appears so simple to us now that we ignore its true merit.

Albert Einstein (1879-1955), One of the greatest scientists, philosophers, Nobel Laureate (1921-22) for his 'Theory of Relativity'. We owe a lot to Indians who taught us how to count, without which no worthwhile scientific discovery could have been made.

André Weil (1906-1998), French, fell in love with mathematics and Sanskrit literature; by the age of sixteen he had read the Bhagavad Gita in the original Sanskrit, wrote in 1884: “What would have been Fermat’s astonishment, if some missionary, just back from India, had told him that his problem had been successfully tackled by native mathematicians almost six centuries earlier”. In fact, Brahmagupta (born 30 BC) and Bhaskara II (working 486) had addressed themselves to some of Fermat’s problems long before they were thought of in the west, and had solved them thoroughly. They have not held a proper place in mathematical history, or received credit for their problems and methods of solution.

Ancient astronomical texts such as Surya Siddhanta and its commentaries contain:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$
$$\cos A \sin c = \cos a \cos b - \sin a \cos b \cos C$$
$$\frac{\sin a}{\sin A} = \frac{\sin b}{\sin B} = \frac{\sin c}{\sin C},$$

where A, B, C are the angles of a spherical triangle, of which the opposite sides are a, b, and c, respectively.

Bhaskara I (before 123 BC) gave the following point-wise approximation formula for $\sin x$, which has a relative error less than 1.9%

$$\sin x \ = \ \frac{16x(\pi - x)}{[5\pi^2 - 4x(\pi - x)]}.$$

Daivajna Varahamihira (working 123 BC) gave trigonometric formulae that correspond to

$$\sin x = \cos(\pi/2 - x), \ \sin^2 x + \cos^2 x = 1 \quad \textbf{and} \quad (1 - \cos 2x)/2 = \sin^2 x.$$

Bhaskara II developed spherical trigonometry systematically and discovered many trigonometric results, such as

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b.$$

Important exact formula for finding the area A of a triangle having sides a, b, and c, known in the literature as Heron's (Heron of Alexandria, about 75 AD), was stated earlier by Bhaskara I:

$$A = \sqrt{s(s-a)(s-b)(s-c)},$$

where $2s = a+b+c$. This formula was later generalized by Brahmagupta for finding the area A of any cyclic quadrilateral given its four sides a, b, c, d as

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)},$$

where $2s = a+b+c+d$, which was later rediscovered by Willebrord Snell (1580–1626).

The term tangent was coined by Leibniz in 1684, which has been credited solely to Newton and Leibniz. For Newton, the calculus was geometrical, while Leibniz took it toward analysis. However, the concept of a derivative had been developed almost 1200 years before Newton and Leibniz by Bhaskara II, who provided differentiation of the trigonometric functions, for example (in modern notation), he established $\delta(\sin x) = \cos x \cdot \delta x$. **Bhaskara II also gave a statement of Michel Rolle's (1652-1719) theorem, concluded that the derivative vanishes at a maxima, and introduced the concept of the instantaneous motion of a planet in his collection** *Siddhanta Siromani*. **Then, in the 14th century, Madhava of Sangamagramma (1340-1425) invented the ideas underlying the infinite series expansions of functions, power series, the trigonometric series of sine, cosine, tangent, and arctangent (these series have been credited to James Gregory (1638-1675), Brook Taylor (1685-1731), and Newton), rational approximations of infinite series, tests of convergence of infinite series, the estimate of an error term, and early forms of differentiation and integration.**

Madhava fully understood the limit nature of the infinite series. This step has been called the "decisive factor onward from the finite procedures of ancient mathematics to treat their limit–passage to infinity", which is in fact the kernel of modern classical analysis. Parameshvara Namboodri (around 1370-1460), a disciple of Madhava, stated an early version of the mean value theorem in his *Lilavathi Bhasya*. This is considered to be one of the most important results in differential calculus, and was later essential in proving the fundamental theorem of calculus, which shows the inverse character of tangent and area problems. Evangelista Torricelli (1608–1647) was the first to understand the fundamental theorem of calculus geometrically, and this was extended by Gregory while Barrow established a more generalized version, and finally Newton completed the mathematical theory.

Nilakanthan Somayaji (around 1444-1544), following the footsteps of Madhava and his father Parameshvara, provided a derivation and proof of the arctangent trigonometric series and gave the relationship between the power series of π and arctangent, namely,

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \cdots,$$

which in the literature has been credited to Gregory and Leibniz. Guillaume Francois Antoine de L'Hôpital (1661-1704) is known as the author of the world's first text book on differential calculus, but Jyesthadevan (around 1500-1600) wrote the calculus text *Yuktibhasa* in Malayalam (a regional language of the Indian state of Kerala) almost 150 years earlier.

In mathematics a set is defined as a collection of objects or elements, and set theory is considered to be a branch of mathematical logic. It has been documented that Georg Cantor (1845–1918) introduced set theory to the mathematical world in 1874. However, during 200–875 AD, the Jain School of Mathematics in India utilized the concept of sets. In their work, the Jains introduced several different types of sets, such as cosmological, philosophical, karmic, finite, infinite, transfinite, and variable sets. They called the largest set an omniscient set, and the set containing no elements was known as the null set. They also defined the concept of a union of sets and used the method of one–to–one correspondence for the comparison of transfinite sets. In order to determine the order of comparability of all sets, they considered fourteen types of monotone sequences. Thus, Cantor alone is not the founder of set theory. His major contribution was the mathematical systematization of set theory, now known as naive (non–axiomatic) set theory, and the modern understanding of infinity.

Today, set theory is woven into the fabric of modern mathematics. In fact, the language of set theory is used to precisely define nearly all mathematical objects.

The word 'mathematics' was coined by Pythagoras (around 582-481 BC). It meant 'a subject of instruction', and its first part, 'math', comes from an old Indo–European root that is related to the English word 'mind'. The Pythagoreans grouped arithmetic, astronomy, geometry, and music together and for several centuries mathematics referred to only these four subjects.

After Pythagoras several different definitions of mathematics have been proposed. Each one tries to define mathematics with a specific context in mind. For example:

Mathematics is like draughts (checkers) in being suitable for the young, not too difficult, amusing, and without peril of the state. (Plato)

Mathematics is the study of "quantity". (Aristotle)

Mathematics is the gate and key of the sciences, which the saints discovered at the beginning of the world ... and which has always been used by all the saints and sages more than all the sciences. Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world. And what is worse, men who are thus ignorant are unable to perceive their own ignorance and so do not seek a remedy. (Roger Bacon, 1214–1294)

Mathematics is the science of order and measure. (Descartes)

Mathematics–the unshaken foundation of science, and the plentiful fountain of advantage to human affairs. (Barrow)

Mathematics is concerned only with the enumeration and comparison of relations. (Gauss)

Mathematics is the science of what is clear by itself. (Carl Guslov Jacob Jacobi, 1804–1851)

Mathematics is the science which draws necessary conclusions. (Benjamin Peirce, 1809–1880)

Mathematics seems to endow one with something like a new sense. (Charles Robert Darwin, 1809–1882)

Mathematics is the work of the human mind, which is destined rather to study than to know, to seek the truth rather than to find it. (Evariste Galöis, 1811–1832)

Mathematics is not (as some dictionaries today still assert) merely "the science of measurement and number", but, more broadly, any study consisting of symbols along with precise rules of operation upon those symbols, the rules being subject only to the requirement of inner consistency. (George Boole, 1815–1864)

"Do you know what a mathematician is" Lord Kelvin (1824–1907) once asked a class. He stepped to the board and wrote

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$

Putting his finger on what he had written, he turned to the class. "A mathematician is one to whom that is as obvious as that twice two makes four is to you".

Mathematics is a branch of logic. (Julius Wilhelm Richard Dedekind, 1831–1916)

Mathematics is the science of quantity. (Charles Sanders Peirce, 1839–1914)

Mathematics is the science of self–evident things. (Felix Christian Klein, 1849–1925)

Mathematics is the development "of all types of formal, necessary and deductive reasoning". (Alfred North Whitehead, 1861–1947)

Mathematics is a game played according to certain rules with meaningless marks on paper. (David Hilbert, 1862–1943)

Mathematics is a subject identical with logic. (Russell)

Pure mathematics is the class of all propositions of the form 'p implies q' where p and q are propositions. (Russell)

The whole of mathematics is nothing more than a refinement of everyday thinking. (Einstein)

Mathematics is the art of problem solving. (George Polya, 1887–1985)

Mathematics is a development of thought that had its beginning with the origin of man and culture a million years or so ago. To be sure, little progress was made during hundreds of thousands of years. (Leslie Alvin White, 1900–1975)

Mathematics is a spirit of rationality. It is this spirit that challenges, simulates, invigorates and drives human minds to exercise themselves to the fullest. It is this spirit that seeks to influence decisively the physical, normal and social life of man, that seeks to answer the problems posed by our very existence, that strives to understand and control nature and that exerts itself to explore and establish the deepest and utmost implications of knowledge already obtained. (Morris Kline, 1908–1992)

Mathematics today is the instrument by which the subtle and new phenomena of nature that we are discovering can be understood and co-ordinated into a unified whole. In this some of the most advanced and newest branches of mathematics have to be employed and contact with an active school of mathematics is therefore great asset for theoretical physicists. (Homi Jehangir Bhabha, FRS, 1909–1966)

Mathematics is like a chest of tools, before studying the tools in detail, a good workman should know the object of each, when it is used and what it is used for. (Walter Warwick Sawyer, 1911–2008)

Mathematics has also been defined as follows: mathematics is a human creation; mathematics is a natural part of man's cultural heritage; mathematics is the accumulation of human wisdom in an effort to understand and harness the physical, social, and economic worlds; mathematics is a language and a language can be learned only by continuously using it; mathematics is a tool that ideally permits mediocre minds to solve complicated problems expeditiously; mathematics is the study of quantity, structure, space, and change; mathematics is the measurement, properties, and relationships of quantities and sets using numbers and symbols; mathematics is something that man himself creates, and the type of mathematics he works out is just as much a function of the cultural demands of the time as any of his other adaptive mechanisms; mathematics is the science which uses easy words for hard ideas; and, mathematics seeks regularities and pattern in behavior, motion, number, or shape, or even in the substrata of chaos.

A variety of quips and clichés can also define mathematics: "It's an art", "it's a science – in fact, it's the queen and servant of science", "it's what I use when I balance my checkbook", "a game that we play with rules we're not quite sure of", and the apologetic favorite "something I was never good at", and the list can go on and on.

About mathematics several interesting positive views also have been proposed. Perhaps from these we can find some more definitions of mathematics.

Mathematics gives life to her own discoveries; she awakens the mind and purifies the intellect; she brings light to our intrinsic ideas, she abolishes the oblivion and ignorance which are ours by birth. (Proclus Diadochus, 410–485 AD)

Neglect of mathematics works injury to all knowledge, since he who is ignorant of it cannot know the other sciences or the things of this world. (Roger Bacon)

Nature is written in mathematical language. (Galileo Galilei, 1564–1642)

Mathematicians are like Frenchmen; whatever you say to them they translate into their own language, and forthwith it is something entirely different. (Johann Wolfgang von Goethe, 1749–1832)

Mathematics is one of the oldest of sciences; it is also one of the most active; for its strength is the vigor of perpetual youth. (Andrew Russell Forsyth, 1858–1942)

In mathematics I can report no deficiency, except it be that men do not sufficiently understand the excellent use of the Pure Mathematics. (Francis Bacon, 1909–1992)

Yet another positive view of mathematics and hence of mathematicians follows. Mathematicians are of two types: the 'we can' men believe (possibly subconsciously) that mathematics is a purely human invention; the 'there exists' men believe that mathematics has an extra-human *existence* of its own, and that 'we' merely come upon the *eternal truths* of mathematics in our journey through life, in much the same way that a man taking a walk in a city comes across a number of streets with whose planning he had nothing whatever to do (Bell, Men of Mathematics).

We also encounter some negative views about mathematics, such as the idea that "the mathematics that is certain does not refer to reality and mathematics that refers to reality is not certain" (Albert Einstein). Sir William Hamilton (1788-1856), the famed Scottish philosopher, logician, and meta–physicist, viewed mathematics in a way that may be construed as a cruel attack on mathematics and hence on mathematicians: "Mathematics freeze and parch the mind", "an excessive study of mathematics absolutely incapacitates the mind for those intellectual energies which philosophy and life require", "mathematics cannot conduce to logical habits at all", "in mathematics dullness is thus elevated into talent, and talent degraded into incapacity", "mathematics may distort, but can never rectify, the mind". (Bell, Men of Mathematics).

The following parable of "The Blind Men and the Elephant" (a Story from the *Buddhist Sutra*) is relevant to our attempt to define mathematics. Several prominent citizens were engaged in a hot argument about God and the different religions, and could not come to an agreement. So they approached Lord Gautama Buddha (1887–1807 BC) to find out what exactly God looks like. Buddha asked his disciple to get a large majestic elephant and four blind men. He then brought the four blind men to the elephant and told them to find out what the elephant would "look" like. The first blind man touched the elephant's leg and reported that it "looked" like a pillar.

The second blind man touched its tummy and said that an elephant was an inverted ceiling. The third blind man touched the elephant's ear and said that it was a piece of cloth. The fourth blind man held on to the tail and described the elephant as a piece of rope. And all of them ran into a hot argument about the "appearance" of an elephant. The Buddha asked the citizens: "Each blind man had touched the elephant but each of them gives a different description of the animal. Which answer is right?" "All of them are right," was the reply. "Why? Because everyone can only see part of the elephant. They are not able to see the whole animal. The same applies to God and to religions. No one will see Him completely." By this story, Lord Buddha teaches that we should respect all legitimate religions and their beliefs.

This famous "blind men" episode is not meant to disrespect any mathematician. We state it only to point out that "*any definition of mathematics, however elaborate or epigrammatic, will fail to lay bare its fundamental structure and the reasons for its universality***" (Hermann Klaus Hugo Weyl, 1885–1955). In fact, the definition of mathematics continues to change with time and innovation.**

A PROOF is an explanatory note that the human senses can perceive and that leads the mind and intellect to establish the validity of a fact or argument. Proofs express subjective truth, not universal truth. Since humans have limited perceptive power, there are innumerable subtle assumptions that must be made during the process of proof which are beyond human intellect, therefore any proof can never be exact. This is why we accept whatever truth can be established within our human limitations. Yet the great Sages and Rishis of history possessed (in Samadhi Stage) extraordinarily subtle insight as well as unique experiences, which led them to establish certain truths. These are known as empirical theories, and are simply well-founded empirical generalizations or laws about the properties and behaviors of objects that are obtained by examining a large number of instances and seeing that they conform without exception to a single general pattern. These theories could never be questioned and have withstood the test of time. One accepts such theories because of his total faith in the wisdom of these sages, or because he believes this to be the primary (or the sole) path to knowledge.

It should be noted that proof has different shades of meaning depending upon the field in which it is considered. For example, within the legal system, the burden of proof in a civil court is "Preponderance of evidence" whereas in a criminal court it is, "Beyond a reasonable doubt"; in the scientific world (Physics, Chemistry, Engineering) truth is established by experiments; in the business world, statistical analyses of data are performed and conclusions are derived; in political or international matters, organizations or group of individuals are entrusted with finding the 'truth' of a given issue and they come up with their conclusion based on classified intelligence reports, site visits, interviews, and other such activities; in a religious sense, proof is difficult to even describe. In scientific measurements, due to uncertainty, the absolute truth can not be found although we may get extremely close to doing so.

A proof of a mathematical statement is a logical argument that shows the statement is true according to certain accepted standards. It is believed that the idea of proving a statement began about the 5th century BC in Greece. Initially, philosophers developed a way of convincing each other of the truth of particular mathematical statements. They used definitions and axioms (postulates) to do this convincing. Over time, these ideas were collected and organized in thirteen books by Euclid (325-263 BC) called, “Elements of Mathematics” Recent research on Indian mathematics shows that there were other methods of investigating mathematical ideas other than commonly accepted proof methods as described by Euclid.

In fact, Indian mathematicians wrote Commentaries on the works of earlier writers to elaborate ideas, to find other ways of achieving results and of making new and useful applications. The word Upapattis (in Pali, Sanskrit, and Marathi languages) is used for 'proof'. Following are some of the important features of upapattis in Indian mathematics.

1. **The Indian mathematicians are clear that results in mathematics, even those enunciated in authoritative texts, cannot be accepted as valid unless they are supported by yukti or upapatti. It is not enough that one has merely observed the validity of a result in a large number of instances.**

2. **Several commentaries written on major texts of Indian mathematics and astronomy present upapattis for the results and procedures enunciated in the text.**
3. **The upapattis are presented in a sequence proceeding systematically from known or established results to finally arrive at the result to be established.**
4. **In the Indian mathematical tradition the upapattis mainly serve to remove doubts and obtain consent for the result among the community of mathematicians.**
5. **The upapattis may involve observation or experimentation. They also depend on the prevailing understanding of the nature of the mathematical objects involved.**

6. The method of tarka or "proof by contradiction" is used occasionally. But there are no upapattis which purport to establish existence of any mathematical object merely on the basis of tarka alone. In this sense the Indian mathematical tradition takes a "constructivist" approach to the existence of mathematical objects.
7. The Indian mathematical tradition did not subscribe to the ideal that upapattis should seek to provide irrefutable demonstrations establishing the absolute truth of mathematical results.
8. There was no attempt made in Indian mathematical tradition to present the upapattis in an axiomatic framework based on a set of self-evident (or arbitrarily postulated) axioms which are fixed at the outset.
9. While Indian mathematicians made great strides in the invention and manipulation of symbols in representing mathematical results and in facilitating mathematical processes, there was no attempt at formalization of mathematics.

It should be noted that in all exiting literature no mention has been made about the contribution made by Indian mathematicians or the fact that these mathematicians viewed the concept of proof differently than what was described by Euclid.

There is a widespread notion that once something has been proved mathematically, then it is, as it were, set in stone that we have a mathematical proof that remains 'true for all time'. "Not so", says James R. Meyer. According to him "A proof is always dependent on whatever system of assumptions and rules of inference used to create the proof." He writes that most mathematical proofs that anyone will encounter fall a long way short of idealistic concepts.

While the definitions of Mathematics and Proof continue to change with time and innovation. Mathematics rewards its creator with a strong sense of aesthetic satisfaction. It helps us understand man's place in the universe and enables us to find order in chaos. Under certain axioms, mathematics is the most absolute, ever lasting, precise, significant, and universal subject. It is perceived as the highest form of thought in the world of learning. No doubt, mathematics is one of the greatest creations of mankind–if it is not indeed the greatest. Mathematics will live forever.

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